Exercise 41

Find the derivative of the function.

$$f(t) = \sin^2(e^{\sin^2 t})$$

Solution

Take the derivative using the chain rule.

$$\begin{aligned}
f'(t) = \frac{df}{dt} &= \frac{d}{dt}\left[\sin(e^{\sin^2 t})\right]^2 \\
&= 2\left[\sin(e^{\sin^2 t})\right] \cdot \frac{d}{dt}\left[\sin(e^{\sin^2 t})\right] \\
&= 2\left[\sin(e^{\sin^2 t})\right] \cdot \left[\cos(e^{\sin^2 t})\right] \cdot \frac{d}{dt}(e^{\sin^2 t}) \\
&= 2\left[\sin(e^{\sin^2 t})\right] \cdot \left[\cos(e^{\sin^2 t})\right] \cdot (e^{\sin^2 t}) \cdot \frac{d}{dt}(\sin^2 t) \\
&= 2\left[\sin(e^{\sin^2 t})\right] \cdot \left[\cos(e^{\sin^2 t})\right] \cdot (e^{\sin^2 t}) \cdot (2\sin t) \cdot \frac{d}{dt}(\sin t) \\
&= 2\left[\sin(e^{\sin^2 t})\right] \cdot \left[\cos(e^{\sin^2 t})\right] \cdot (e^{\sin^2 t}) \cdot (2\sin t) \cdot (\cos t) \\
&= e^{\sin^2 t} \sin(2e^{\sin^2 t}) \sin 2t
\end{aligned}$$